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THE HYPERSONIC WAKE PROGRAM
I. A FORTRAN PROGRAM FOR THE INTEGRAL SOLUTION

by

James E. Wollrab

May 1966

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U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

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I. A FORTRAN PROGRAM
FOR THE INTEGRAL SOLUTION

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ABSTRACT

A FORTRAN program is explicitly listed for the basic integral solution of the wake of a hypersonic blunt or conical reentry body. The near wake pressure gradient, nonequilibrium chemistry, and laminar-turbulent transition are included.

CONTENTS

SYMBOLS

Text	Program*	Definition
a		Base area, feet^2, Equation (70).
A', A''		Constants, Equations (60) and (61).
	ALT	Altitude, feet
C_D	CD	Drag coefficient, Equation (70).
$\bar{c}_p$		Specific heat at constant pressure of air mixture, °K, Equation (15).
$c_p^{(i)}$		Specific heat of species i, °K, Equation (15).
D		Drag, pounds, Equation (69).
$D_{ij_{ave}}$		Average binary diffusion coefficient between species i and j, Equation (8).
d	LD	Base diameter of body, feet.
E		Electronic energy, feet^2 per second^2, Equation (54).
E_{chem}		Zero-point energy, feet^2 per second^2, Equation (54).
e_{int}		Internal energy, feet^2 per second^2 per unit mass, Equation (4).
$E_{vib}^{(i)}$		Vibrational energy, feet^2 per second^2 (equal to Planck's constant times $\gamma^{(i)}$).
g		Electronic degeneracy, Equation (54).
H		Total enthalpy, feet^2 per second^2, Equation (57).

*See footnote at end of symbols.

Text	Program*	Definition
h	LHXN (h(x, n))	Enthalpy, feet² per second², Equation (53).
H_{nn_0}	HNNO	$(\partial^2 H/\partial n^2)_0$, Equation (43).
$\tilde{I}$		Unit tensor, Equation (6).
K		Thermal conductivity, Equation (7).
k		Boltzmann's constant, Equation (54).
Le	LEO (Le_0)	Lewis number, Equation (10).
M	W	Molecular weight of atmosphere model, Equation (74).
M_∞	MIN	Free-stream Mach number, Equation (64).
m		Transformation variable, Equation (23).
$M^{(i)}$	FM	Mass of i^{th} species, Equation (54).
n		Transformed radial coordinate, Equation (24).
p	PRESS	Pressure, pounds per foot², Equation (6).
Pr	PRO (Pr_0)	Prandtl number, Equation (10).
q		Dynamic pressure, pounds per foot², Equation (70).
$\overline{q}$		Heat flux vector, Equation (4).
R		Radial distance to shock, feet, Equation (69).
r		Radial coordinate, feet, Equation (25).
T	TEMP	Temperature (kinetic), °K Equation (75).
T_m	TEMPM	Molecular-scale temperature, Equation (76).

*See footnote at end of symbols.

Text	Program*	Definition
t		Time, seconds.
u		Velocity in downstream (x) direction, feet per second, Equation (9).
u_∞	UIN	Free-stream velocity, feet per second.
V		Diffusion velocity, feet per second, Equation (5).
v		Velocity in the radial (r) direction, feet per second, Equation (9).
$\vec{v}$		General fluid velocity, feet per second,vector, Equation (1).
x		Downstream coordinate, feet, Equation (9).
x_t	XT	Laminar turbulent transition coordinate, feet, Equation (64).
Δx		Increment of x, feet.
x_c	START X	Initial x value, feet.
	STOP X	Terminal x value, feet.
X_i		End-point altitudes, feet, Equation (76).
Y	Y	Transition coordinate parameter, Equation (64).
Y_i		End-point temperatures, Equation (77).
Z_0	ZOE_1	Compressibility factor on the axis.
A, B, C, D, E, F, K, L, N, P		Constants (Paragraph 4).
B, C, D, E_i, H_j		Atmosphere constants, Equations (77) through (81).
α	ALPH	Species concentration, Equation (11).

*See footnote at end of symbols.

Text	Program*	Definition
δ	DEL	Radial distance in x, r coordinate system, feet, Equation (72).
δ_m	DELM2 (δ_m^2)	Transformed distance to outer edge of wake, feet, Equation (25).
θ	THETA	Momentum thickness, feet, Equation (17).
θ_E		Energy thickness, feet, Equation (18).
$\theta^{(i)}$		Concentration thickness, feet, Equation (19).
Λ_c	DELTAC	Constant depending on initial conditions, Equation (44).
$\Lambda_e^{(i)}$		Defined by Equation (59).
μ	MU	Viscosity coefficient, $\frac{\text{lb-sec}}{\text{ft}^2}$, Equation (6).
$\gamma^{(i)}$		Vibrational frequency of i^{th} molecular species, Equation (54).
ρ	DENSG	Density, Equation (1), $\frac{\text{lb-sec}^2}{\text{ft}^4}$
ϕ_N	PHI	Cone half-angle, degrees.
ϕ	S2B	Shock half-angle, degrees, Equation (67).
ϕ	. . .	Coordinate in initial cylindrical coordinate system (not used in equations).

SUBSCRIPTS AND SUPERSCRIPTS

i	I	Index for eight chemical species.
c	C	Initial x coordinate for calculation.

*See footnote at end of symbols.

Text	Program*	Definition
e	E	Values at n = 1.
0	0	Values at n = 0.
∞	IN	Free-stream conditions.

*Only a few representative program symbols are listed; however, unlisted symbols can be deduced from the text equations. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text unless noted otherwise.

1. Introduction

The analysis presented in this report is essentially a programming of the hypersonic axisymmetric wake problem as formulated by Bloom and Steiger [1] and Ness and Fanucci. [2] The object of this work is to use the fundamental wake analysis as a basis for an extension of the problem to include contributions from ablation and base mixing and to determine their effects on the initial wake conditions and concentrations of radiating species. The FORTRAN program is in Appendix A. To be complete and internally consistent, the program is described in detail beginning with the basic conservation equations. The solutions are obtained for an air model of eight species using nonequilibrium chemistry. The downstream pressure gradient in the wake and the laminar-turbulent transition are included. However, a number of important assumptions have been employed to make the analysis tenable. The units used in the program are feet, pounds, seconds, and degrees Kelvin. Corresponding units are assumed throughout the text, unless noted otherwise.

2. Definition of the Flow Field

The basic regions of the flow field accompanying a hypersonic body as it enters the atmosphere have been extensively studied both experimentally and theoretically. Air is heated irreversibly and compressed by the strong detached bow shock associated with a blunt body. The fluid entering the boundary layer overexpands around the rear edge of the body. In this base-flow region, the fluid is recompressed and turned back parallel to the axial direction producing a secondary shock which is weaker than the main bow shock. Fluid which is not captured by the recirculating base flow passes through a high pressure, minimum diameter neck and into the viscous wake. A conical body is characterized by an attached shock which produces much less heating in the inviscid portion of the wake and by a thicker boundary layer.

The inner viscous wake may be divided into two regions distinguished by the presence of an axial pressure gradient. The near wake extends from the vicinity of the neck to the downstream coordinate at which the pressure has essentially decayed to the ambient value. The far wake is then controlled primarily by diffusion processes. Large pressure gradients and high temperatures characterize the initial portions of the near wake. The outer inviscid wake passes through the bow shock and the recompression shock to be "swallowed" by the radial growth of the inner wake at some downstream coordinate determined by the flight

characteristics and the laminar-turbulent transition point of the inner wake.

3. Basic Conservation Equations, Transformations, and Boundary Conditions

When both the chemical and fluid-mechanical properties of the flow are considered, conservation relations can be written for the momentum, energy, and chemical species. In addition, an equation expressing global continuity (conservation of mass) is valid. Using concise vector notation, the continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overline{v}) = 0. \tag{1}$$

Because the calculations are done under the assumption of steady flow, all time derivatives vanish.

$$\nabla \cdot (\rho \overline{v}) = 0 \tag{2}$$

Similarly, the momentum, energy, and species equations are respectively

$$\nabla \cdot (\rho \overline{v}\overline{v} + \widetilde{p}) = 0 \tag{3}$$

$$\nabla \cdot (\tfrac{1}{2} \rho |\overline{v}|^2 \overline{v} + \rho e_{int} \overline{v} + \overline{v} \cdot \widetilde{p} + \overline{q}) = 0 \tag{4}$$

$$\nabla \cdot \rho^{(i)} \left(\overline{v} + \overline{V}^{(i)}\right) = \left(\frac{\partial \rho^{(i)}}{\partial t}\right)_{chem} \tag{5}$$

$\widetilde{p}$ is the pressure tensor, $\overline{q}$ is the heat flux vector, and $\overline{V}^{(i)}$ is the diffusion velocity.

$$\widetilde{p} = p\widetilde{I} - \mu\left[\nabla \overline{v} + (\nabla \overline{v})^{\dagger} - \tfrac{2}{3} (\nabla \cdot \overline{v})\widetilde{I}\right] \tag{6}$$

$$\overline{q} = -K\nabla T + \sum_i \rho^{(i)} \overline{V}^{(i)} h^{(i)} \tag{7}$$

$$\mathbf{V}^{(i)} = -D_{ij_{ave}} \nabla \alpha^{(i)}/\alpha^{(i)} \tag{8}$$

Superscript i represents the i^{th} chemical species of the reacting air mixture. $D_{ij_{ave}}$ is the average binary diffusion coefficient between species i and j. $\mathbf{I}$ represents the unit tensor, K is the thermal conductivity, and e_{int} represents the internal energy per unit mass of the air mixture. The remaining notation can be obtained from the list of symbols.

The physical coordinate system consists of a downstream coordinate x, a radial coordinate r, and an angular coordinate ϕ. However, the cylindrical symmetry of the problem removes any ϕ dependence. It is also possible to apply boundary layer approximations if the gradients normal to the flow direction are stronger than those along the flow direction.

a. Momentum

$$\frac{\partial}{\partial x}(r\rho u^2) + \frac{\partial}{\partial r}(r\rho uv) - \frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial x}(rp) = 0 \tag{9}$$

b. Energy

$$\frac{\partial}{\partial x}(r\rho uH) + \frac{\partial}{\partial r}(r\rho vH) = \frac{\partial}{\partial r}\left(\frac{\mu}{Pr}\, r\, \frac{\partial H}{\partial r}\right) + \frac{\partial}{\partial r}\left[\frac{\mu}{Pr}(Pr-1)ru\frac{\partial u}{\partial r}\right] + \frac{\partial}{\partial r}\left[\frac{\mu}{Pr}(Le-1)r\sum_i h^{(i)}\,\frac{\partial \alpha^{(i)}}{\partial r}\right] \tag{10}$$

c. Chemical Species

$$\frac{\partial}{\partial x}\left(r\rho u\alpha^{(i)}\right) + \frac{\partial}{\partial r}\left(r\rho v\alpha^{(i)}\right) - \frac{\partial}{\partial r}\left(r\mu\frac{Le}{Pr}\,\frac{\partial \alpha^{(i)}}{\partial r}\right) = \left(r\frac{\partial \rho^{(i)}}{\partial t}\right)_{chem} \tag{11}$$

d. Continuity

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial r}(r\rho v) = 0 \tag{12}$$

Using the expression for the temperature gradient

$$\nabla T = \frac{1}{\overline{c}_p}\nabla\left(H - \frac{u^2}{2}\right) - \frac{1}{\overline{c}_p}\sum_i h^{(i)}\nabla a^{(i)} \tag{13}$$

and Equation (8) for the diffusion velocity, the heat flux vector may be written as

$$\overline{q} = -\frac{\mu}{Pr}\nabla H + \frac{\mu}{Pr}\nabla\left(\frac{u^2}{2}\right) + \frac{\mu}{Pr}(1 - Le)\sum_i h^{(i)}\nabla a^{(i)} \tag{14}$$

$$\overline{c}_p = \sum_i a^{(i)} c_p{}^{(i)}. \tag{15}$$

The conservation equations can now be integrated across the wake from $r = 0$ to $r = \delta$. $\delta(x)$ represents the wake thickness as a function of the downstream coordinate x. Using Leibnitz's rule for the derivative of an integral, the continuity equation becomes

$$(r\rho v)_e = (r\rho u)_e \frac{d\delta}{dx} - \frac{d}{dx}\int_0^\delta r\rho u dr. \tag{16}$$

Further defining the integrals

$$\theta = \int_0^\delta \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right) r dr \tag{17}$$

$$\theta_E = \int_0^\delta \frac{\rho u}{\rho_e u_e}\left(1 - \frac{H}{H_e}\right) r dr \tag{18}$$

$$\theta^{(i)} = \int_0^\delta \frac{\rho u}{\rho_e u_e}\left(a_e{}^{(i)} - a^{(i)}\right) r dr \tag{19}$$

the integral form of the momentum, energy, and chemical species equations in the x, r system is

$$\frac{d\theta}{dx} + \theta \frac{d}{dx}\left(\ell n \rho_e u_e^2\right) - \left(\int_0^{\delta} \frac{\rho u}{\rho_e u_e} rdr\right) \frac{d}{dx} \ell n\, u_e$$

$$- \frac{\delta^2}{2\rho_e u_e^2} \frac{dp}{dx} = 0 \tag{20}$$

$$\rho_e u_e \theta_E = \text{constant} \tag{21}$$

$$\frac{d\theta^{(i)}}{dx} + \theta^{(i)} \frac{d}{dx} (\ell n\, \rho_e\, u_e) - \left(\int_0^{\delta} \frac{\rho u}{\rho_e u_e} rdr\right) \frac{d\alpha_e^{(i)}}{dx} =$$

$$- \frac{1}{\rho_e u_e} \int_0^{\delta} \left(\frac{\partial \rho^{(i)}}{\partial t}\right)_{chem} rdr . \tag{22}$$

The transformed radial coordinate n is now introduced by the Dorodnitzen transformation through the relationships

$$mdm = \frac{\rho}{\rho_e} rdr \tag{23}$$

$$m = n\,\delta_m \tag{24}$$

which lead to

$$\delta_m^2\, ndn = \frac{\rho}{\rho_e} rdr . \tag{25}$$

The effect of this transformation is to remove the radial density dependence from the calculations by introducing the x, n coordinate system in place of the cylindrical x, r system.

Several sets of boundary conditions apply to the concentrations, velocity, and enthalpy at the axis (n = 0) and on the outer edge (n = 1).

$n = 0$:

$$\alpha^{(i)} = \alpha_0^{(i)}(x) \; ; \left(\frac{\partial \alpha^{(i)}}{\partial n}\right)_0 = 0 \tag{26}$$

$$u = u_0 (x) \; ; \left(\frac{\partial u}{\partial n}\right)_0 = 0 \tag{27}$$

$$H = H_0 (x) \; ; \left(\frac{\partial H}{\partial n}\right)_0 = 0 \tag{28}$$

$n = 1$:

$$\alpha^{(i)} = \alpha_e^{(i)}(x) \; ; \left(\frac{\partial \alpha^{(i)}}{\partial n}\right)_e = \left(\frac{\partial^2 \alpha^{(i)}}{\partial n^2}\right)_e = 0 \tag{29}$$

$$u = u_e (x) \; ; \left(\frac{\partial u}{\partial n}\right)_e = \left(\frac{\partial^2 u}{\partial n^2}\right)_e = 0 \tag{30}$$

$$H = H_e \; ; \left(\frac{\partial H}{\partial n}\right)_e = \left(\frac{\partial^2 H}{\partial n^2}\right)_e = 0 \tag{31}$$

These boundary conditions follow from a consideration of the physical model of the symmetrical wake. From these conditions it is possible to formulate profile expressions of the proper form

$$\frac{\alpha^{(i)}(x, n) - \alpha_0^{(i)}(x)}{\alpha_e^{(i)}(x) - \alpha_0^{(i)}(x)} = \frac{u(x, n) - u_0 (x)}{u_e (x) - u_0 (x)} = 6n^2 - 8n^3 + 3n^4. \tag{32}$$

For the enthalpy, $\left(\frac{\partial^2 H}{\partial n^2}\right)_0$ is included by using a fifth-degree polynomial.

$$\begin{aligned} H = H_0 &+ (H_e - H_0)(10n^3 - 15n^4 + 6n^5) \\ &+ \tfrac{1}{2} H_{nn_0} (n^2 - 3n^3 + 3n^4 - n^5) \end{aligned} \tag{33}$$

The momentum Equation (9), the energy Equation (10), and the chemical species Equation (11) are next specialized to $n = 0$ and $n = 1$ after transformation to the x,n system.

n = 0:

$$\frac{du_0}{dx} = \frac{24\mu_0}{\rho_e u_0 \delta_m^2} (u_e - u_0) - \frac{1}{\rho_0 u_0} \frac{dp}{dx} \tag{34}$$

$$\frac{dH_0}{dx} = \frac{2\mu_0}{\rho_e u_0 \delta_m^2} \left[\frac{H_{nn_0}}{Pr_0} + 12 \left(\frac{Pr_0 - 1}{Pr_0} \right) u_0 (u_e - u_0) \right.$$

$$\left. + 12 \left(\frac{Le_0 - 1}{Pr_0} \right) \left(\sum_i \alpha_e^{(i)} h_0^{(i)} - h_0 \right) \right] \tag{35}$$

$$\frac{d\alpha_0^{(i)}}{dx} = \frac{24\mu_0 \, Le_0}{\rho_e \, u_0 \delta_m^2 \, Pr_0} \left(\alpha_e^{(i)} - \alpha_0^{(i)} \right) + \frac{1}{u_0} \left[\frac{1}{\rho} \left(\frac{2\rho^{(i)}}{2t} \right) \text{chem} \right]_0 \tag{36}$$

n = 1:

$$\frac{du_e}{dx} = - \frac{1}{\rho_e u_e} \frac{dp}{dx} \tag{37}$$

$$\frac{d\alpha_e^{(i)}}{dx} = \frac{1}{u_e} \left[\frac{1}{\rho} \left(\frac{\partial \rho^{(i)}}{\partial t} \right) \text{chem} \right]_e \tag{38}$$

Because the stagnation enthalpy is assumed to be constant along the outer edge of the wake, the energy equation is automatically satisfied.

The transformed integral momentum equation is

$$\frac{d\theta}{dx} = - \theta \frac{d\ell n \, \rho_e}{dx} + \left[2\theta + \frac{\delta^2}{2} - \frac{\delta_m^2}{10} \left(4 + \frac{u_0}{u_e} \right) \right] \frac{1}{\rho_e u_e^2} \frac{dp}{dx} \tag{39}$$

where

$$\delta_m^2 = \frac{210 \, \theta}{\left(1 - \frac{u_0}{u_e} \right) \left(10 + 11 \frac{u_0}{u_e} \right)} \tag{40}$$

The integral form of the energy Equation (21) remains unchanged and the constant is given by the value of $\rho_e u_e \theta_E$ at the initial coordinate of the calculation.

$$\rho_e u_e \theta_E = (\rho_e u_e \theta_E)_c \tag{41}$$

where

$$\theta_E = \frac{\delta_m^2}{22}\left[\frac{1}{210}\left(1-\frac{H_0}{H_e}\right)\left(349 + 311\frac{u_0}{u_e}\right) - \frac{H_{nn_0}}{840\, H_e}\left(43 + 23\frac{u_0}{u_e}\right)\right] \tag{42}$$

The shape parameter H_{nn_0} is then

$$H_{nn_0} = 4(H_e - H_0)\left[\frac{349\, u_e + 311\, u_0}{43\, u_e + 23\, u_0}\right] - \frac{(\rho_e\, \delta_m^2)_c \quad \Lambda_c}{\rho_e\, \delta_m^2 \quad (43\, u_e + 23\, u_0)} \tag{43}$$

Λ_c depends on the initial conditions as indicated by the subscript c.

$$\Lambda_c = 4\,(H_e - H_{0_c})\,(349\, u_e + 311\, u_0)_c - (H_{nn_0})_c\,(43\, u_e + 23\, u_0)_c \tag{44}$$

4. Reacting Air Mixture

A reacting air mixture containing eight chemical species is used to represent the fluid chemistry. The air species are N_2, O_2, N, O, NO^+, O_2^-, NO, and e^-. The 22 allowed chemical reactions involving these eight species are

(R1 - R5)	$O_2 + X \rightleftharpoons 2O + X$	(X = O, O_2, N, N_2, NO)
(R6 - R10)	$N_2 + X \rightleftharpoons 2N + X$	(X = O, O_2, N, N_2, NO)
(R11 - R15)	$NO + X \rightleftharpoons N + O + X$	(X = O, O_2, N, N_2, NO)
(R16)	$NO + O \rightleftharpoons N + O_2$	
(R17)	$N_2 + O \rightleftharpoons NO + N$	
(R18)	$N + O \rightleftharpoons NO^+ + e^-$	
(R19)	$NO + e^- \rightleftharpoons NO^+ + 2e^-$	
(R20)	$N_2 + O_2 \rightleftharpoons 2NO$	
(R21)	$O_2 + e^- + O_2 \rightleftharpoons O_2^- + O_2$	
(R22)	$NO^+ + O_2^- \rightleftharpoons NO + O_2$	

Constants involved in the chemical kinetics are given by Ness and Fanucci.[2]

The chemical kinetics enter the analvsis through the chemical production terms $\left(\frac{\partial \rho^{(i)}}{\partial t}\right)_{chem}$ in the law of mass action. The partition

functions are derived by assuming equilibrium for all degrees of freedom. The resulting expressions are

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(O)}}{\partial t}\right)_{chem} = A + \frac{m^{(O)}}{m^{(NO)}}(C - D - E) - \frac{m^{(O)}}{m^{(NO^+)}} F \tag{45}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(N)}}{\partial t}\right)_{chem} = B + \frac{m^{(N)}}{m^{(NO)}}(C + D + E) - \frac{m^{(N)}}{m^{(NO^+)}} F \tag{46}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(O_2)}}{\partial t}\right)_{chem} = -A + \frac{m^{(O_2)}}{m^{(NO)}}(D - \tfrac{1}{2} L) - \frac{m^{(O_2)}}{m^{(O_2^-)}}(N - P') \tag{47}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(N_2)}}{\partial t}\right)_{chem} = -B - \frac{m^{(N_2)}}{m^{(NO)}}(E + \tfrac{1}{2} L) \tag{48}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(NO)}}{\partial t}\right)_{chem} = E + L - C - D - K + \frac{m^{(NO)}}{m^{(O_2^-)}} P' \tag{49}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(NO^+)}}{\partial t}\right)_{chem} = F + \frac{m^{(NO^+)}}{m^{(NO)}} K - \frac{m^{(NO^+)}}{m^{(O_2^-)}} P' \tag{50}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(e^-)}}{\partial t}\right)_{chem} = \frac{m^{(e^-)}}{m^{(NO^+)}} F + \frac{m^{(e^-)}}{m^{(NO)}} K - \frac{m^{(e^-)}}{m^{(O_2^-)}} N \tag{51}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho^{(O_2^-)}}{\partial t}\right)_{chem} = N - P'. \tag{52}$$

The constants A, B, C, D, E, F, K, L, N, and P' are given by Ness and Fanucci.[2]

The total static enthalpy h of the air mixture is

$$h = \sum_i \alpha^{(i)} h^{(i)} = H - \frac{u^2}{2} \tag{53}$$

The individual species states enthalpy is a function of the static temperature and molecular and atomic constants only.

$$h^{(i)} = \frac{7}{2}\frac{kT}{m^{(i)}} + \frac{E_{vib}^{(i)}}{m^{(i)}\left(e^{\frac{E_{vib}^{(i)}}{kT}} - 1\right)}$$

$$+ \frac{1}{m^{(i)}}\left[\frac{\sum_{\ell} g_{\ell}^{(i)}\; E_{\ell}^{(i)}\; e^{\frac{-E_{\ell}^{(i)}}{kT}}}{\sum_{\ell} g_{\ell}^{(i)}\; e^{\frac{-E_{\ell}^{(i)}}{kT}}}\right] + \frac{E_{chem}^{(i)}}{m^{(i)}} \tag{54}$$

From the equation of state,

$$p = \rho\; kT \sum_{i} \frac{a^{(i)}}{m^{(i)}} \tag{55}$$

differentiation leads to

$$\frac{d\ell n\; \rho e}{dx} = \frac{dp}{dx} - \frac{d\ell n\; Te}{dx} - \frac{\sum_{i} \frac{1}{m^{(i)}} \frac{d\alpha_e^{(i)}}{dx}}{\sum_{i} \frac{\alpha_e^{(i)}}{m^{(i)}}} \tag{56}$$

The temperature derivative can be obtained from

$$H_e = \sum_{i} \alpha_e^{(i)}\; h_e^{(i)} + \frac{u_e^2}{2} \tag{57}$$

Substitution of the $h_e^{(i)}$ values into Equation (57) and differentiation gives

$$\frac{d\ell n\; T_e}{dx} = - \frac{A' + \frac{A''}{kT_e}}{\sum_{i} \frac{\alpha_e^{(i)}}{m^{(i)}}\left[\left(\Lambda_e^{(i)}\right)^2 + \Lambda_e^{(i)} \frac{E_{vib}^{(i)}}{kT_e} + \frac{7}{2}\right] + \sum_{i} \frac{5}{2} \frac{\alpha_e^{(i)}}{m^{(i)}}} \tag{58}$$

$$\Lambda_e^{(i)} = \frac{\frac{E_{vib}^{(i)}}{kT_e}}{e^{\frac{E_{vib}^{(i)}}{kT_e}} - 1} \tag{59}$$

$$A' = \sum_i \frac{1}{m^{(i)}}\left(\frac{7}{2} + \Lambda_e^{(i)}\right) \frac{d\alpha_e^{(i)}}{dx} + \sum_i \frac{5}{2} \frac{1}{m^{(i)}} \frac{d\alpha_e^{(i)}}{dx} \tag{60}$$

$$A'' = u_e \frac{du_e}{dx} + \sum_i \frac{E_{chem}^{(i)}}{m^{(i)}} \frac{d\alpha_e^{(i)}}{dx} \tag{61}$$

5. Laminar-Turbulent Transition and the Pressure Gradient

The laminar-turbulent transition is treated by changing the viscosity model. The laminar portion of the wake is governed by the Sutherland equation.

$$\mu_0 = 3.05 \times 10^{-8} \frac{T_0^{5/2}}{T_0 + 110} \tag{62}$$

The turbulent viscosity is

$$\mu_0 = 0.02\ \delta_m\ \rho_e\ (u_e - u_0) \tag{63}$$

The transition coordinate is computed using the equation formulated by Zeiberg.[3]

$$x_t = \frac{\mu_\infty Y}{\rho_\infty u_\infty}\left[-\frac{5}{M_\infty^2} + \frac{6\left(1 + \frac{5}{M_\infty^2}\right) M_\infty^2 \sin^2\phi}{M_\infty^2 \sin^2\phi + 5}\left(\frac{6}{7M_\infty^2 \sin^2\phi - 1}\right)^{5/7}\right] \tag{64}$$

Y is defined by

$$M_\infty \leq 8: \quad \log_{10} Y = 4.744 + 0.124 M_\infty + 0.00976\, M_\infty^2 \tag{65}$$

$$M_\infty > 8: \quad \log_{10} Y = 5.48 + 0.11\, M_\infty \tag{66}$$

A cubic can be written for the shock angle ϕ

$$(\sin^2\phi)^3 - \left[\frac{M_\infty^2 + 2}{M_\infty^2} + 1.4 \sin^2\phi_c\right](\sin^2\phi_N)^2 + \left[\frac{2M_\infty^2 + 1}{M_\infty^4} + \left(1.44 + \frac{0.4}{M_\infty^2}\right)\sin^2\phi_N\right]\sin^2\phi - \frac{\cos^2\phi_N}{M_\infty^4} = 0 \tag{67}$$

ϕ_N is the cone half-angle.

For a blunt body (detached bow shock) $\phi_N = \frac{\pi}{2}$ and $\phi = \frac{\pi}{2}$. For a cone, ϕ is obtained as the intermediate root of Equation (67) which has three positive roots for reasonable cone angles.

The pressure gradient in the near wake is expressed by the equation

$$\frac{p}{p_\infty} = \frac{0.0665 M_\infty^2}{x/d} + 1 \tag{68}$$

Initial calculations were begun at a coordinate downstream of the neck $(x > 0)$ to prevent the first term from becoming infinite.

From the momentum theorem the drag can be used to give an initial value for the momentum thickness.

$$D = 2\pi \int_0^R (p_\infty - p)\, r dr + 2\pi \int_0^R \rho u\, (u_\infty - u)\, r dr \tag{69}$$

R is the radial distance to the shock. It is assumed that the total drag occurs in the area up to $R = \delta$ and that $p = p_\infty$ for $R > \delta$.

$$D = c_D\, q\, a = c_D \left(\frac{\rho_\infty u_\infty^2}{2}\right) \frac{\pi d^2}{4} \tag{70}$$

The initial momentum thickness is then

$$\theta_c = \frac{\rho_\infty}{(\rho e)_c} \frac{c_D d^2}{16} + \frac{(p_c - p_\infty)\, \delta_c^2}{2\, (\rho_e)_c\, u_e^2} \tag{71}$$

Since

$$\delta = \sqrt{2}\, \delta_m \left[\int_0^1 \frac{\rho_e(x)}{\rho(x,n)}\, n dn \right]^{1/2}, \tag{72}$$

$$\theta_c = \frac{\dfrac{\rho_\infty}{(\rho e)_c} \dfrac{c_D d^2}{16}}{\left[1 - \dfrac{210\, (p - p_\infty)}{\rho_e u_e^2 \left(1 - \frac{u_0}{u_e}\right)\left(10 + 11 \frac{u_0}{u_e}\right)} \displaystyle\int_0^1 \frac{\rho_e}{\rho}\, n dn \right]_c} \tag{73}$$

6. Sequence of Calculations

A printout of the FORTRAN source program is given in Appendix A. In the right-hand margin references are made to the equations used in the calculations. Important program symbols are defined in the list of symbols and correlated with the symbols used in the text.

The input data include α values for O_2, N_2, O, N, NO, O_2^-, NO^+, and e^- and α_e values for O_2 and N_2. The remaining initial α_e values have been set equal to zero internally. Initial u_0, H_0, T_0, ϕ_n, Z_0, H_{nn_0}, and x values as well as Pr_0, Le_0, c_D, d, u_∞, and final x values are used as input. The altitude must be provided to initiate the atmospheric subroutine for the calculation of free-stream temperature, pressure, density, viscosity, Reynolds number, and speed of sound.[4] This routine was adopted from an IBM 1620 program for the calculation of atmospheric properties up to 2320 kilofeet. A polynomial function of temperature as a function of geopotential altitude is used up to approximately 295 kilofeet. Below this altitude it is assumed that air has a constant molecular weight of 28.9644. Geometric altitude is employed above 295 kilofeet. For these higher altitudes the molecular weight is given by the expression

$$M = 32.154 + AKM\left[-0.034513 + AKM\,(2.0326 \times 10^{-6} + 2.1032 \times 10^{-8}\,AKM)\right] \qquad (74)$$

where AKM is the altitude in kilometers. Molecular-scale temperature (°R), the rate of change of temperature with geopotential altitude, and the initial pressure values ($lb\text{-}ft^{-2}$) are listed for 23 altitude values in the atmosphere table.

The kinetic temperature is obtained from the molecular-scale temperature through the equation

$$T = \frac{T_m M}{28.9644} \qquad (75)$$

where the molecular-scale temperature can be obtained from the relation

$$T_m(X) = D + BX + \sum_i E_i \left| X - X_i \right| \qquad (76)$$

The X_i's are altitudes at the end points of straight line segments over which the variation of altitude and molecular-scale temperature is assumed linear. The constants D, B, and E_i are functions of the X_i and the end-point temperatures which are denoted as Y_i.

$$H_j = -\frac{1}{2}\left(\frac{Y_j - Y_{j-1}}{X_j - X_{j-1}}\right) \qquad (j = 2 \ldots 23) \tag{77}$$

$$D = C + H_2X_1 + H_{23}X_{23} \tag{78}$$

$$C = \frac{Y_1 + Y_{23}}{2} \tag{79}$$

$$B = -H_2 - H_{23} \tag{80}$$

$$E_i = H_j - H_{j+1} \tag{81}$$

Once the free-stream conditions are known, the laminar-turbulent transition coordinate is calculated as a function of the free-stream conditions and M_∞ using Equation (64). The axial viscosity is obtained from the Sutherland Equation (62) fc. the laminar region and from Equation (63) in the turbulent zone. Constants A through P' which appear in the chemical production terms are evaluated on the axis using ρ_0, T_0, and $\alpha_0, \ldots$ and at the edge of the wake using ρ_e, T_e, and $\alpha_e, \ldots$ Density species derivatives in Equations (45) through (52) are then calculated at the axis and on the outer edge. From the profiles for species concentrations, velocity, and enthalpy, $\alpha(x,n)$, $u(x,n)$, and $H(x,n)$ can be evaluated. $T(x,n)$ is then obtained by iteration of Equation (53) using the individual species enthalpies given in Equation (54). Once the pressure is evaluated, from Equation (68), $\rho(x,n)$ is given by Equation (55).

At this point, θ_c is calculated from Equation (73). This is only done for the initial coordinate. Once δ_m is obtained from Equation (40), the derivatives of $\alpha_0^{(i)}$, $\alpha_e^{(i)}$, p, u_0, and u_e in Equations (36), (38), (68), (34), and (37), respectively, allow these parameters to be calculated at the next downstream coordinate. Equations (58) and (56) give $d\ell n\, T_e/dx$ and $d\ell n\, \rho_e/dx$, and δ is obtained by integration of Equation (72). These lead to $d\theta/dx$ in Equation (39). Λ_c and H_{nn_0} from Equations (44) and (43) allow dH_0/dx to be determined using Equation (35).

Finally, T_0 can be evaluated using the new values of H_0, u_0, and $\alpha_0^{(i)}$, and the calculations are then repeated for an incremented value of x. In this calculation, x was incremented uniformly as $\Delta x = 0.1$. Better results may be obtained by using smaller increments for small values of x; however, the error was not large enough to justify a procedure of this type.

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Appendix A

FORTRAN PROGRAM

The hypersonic WAKE program described in the text is given along with the required subroutines. Comments appearing in the right-hand margin refer to equations discussed in the text. Some important symbols are defined in the list of symbols; however, most of the quantities can be identified by reference to the sequence of calculation (Paragraph 6) and the original equations.

```
C     ----HYPERSONIC WAKE CALCULATION ---
C      - - -
      DIMENSION ALPHE(8),ALPHO(8),FM(8),FN(4),ALPHE1(8),ALPHO1(8),H(4),
     1T(4),A(2),B(2),C(2),D(2),E(2),F(2), K(2), L(2),  N(2),PP(2),
     2ALPH(8),TOE(2),RHOE(2),        UOE(2),LHOE(2),HOF(2),ZOF(2),
     3AA(23,4),HH(23),RHOE1(2),UOF1(2),HOE1(2),
     4ZOE1(2),ALPHXN(8,4),UXN(4),HXN(4),LHXN(4),TXN(4),HSUBS(8),RHOEXN(4
     5),PPTSBO(8),PPTSBE(8),FK(4),DALPHO(8),DALPHE(8)
      DIMENSION PPTO(2),PPTN(2),PPTO2(2),PPTN2(2),PPTNO(2),PPTEM(2),
     1PPTO2M(2),PPTNOP(2)
      DIMENSION COE(51),ROOTR(50),ROOTI(50),RUSED(3),BETA(3)
      DATA FM/ 16.,14.,32.,30.,30.,32.,.54171181E-4,28./,EP19,EP20,EP21,
     1EP22/4*1./,R,GA,S/1545.31,32.1741,20855490./
C     ---- -  ** 1962 ATMOSPHERE PROPERTIES ----------
      DATA AA/0.,36088.8,65616.,104985.6,154197.6,170601.6,200128.8,
     1259183.2,291148.,295272.,328080.,360888.,393696.,492120.,524928.,
     2557736.,623352.,754584.,984240.,1312320.,1640400.,1968480.,
     32296560.,518.67,389.97,389.97,411.57,487.17,487.17,454.77,325.17,
     4325.17,325.17,379.17,469.17,649.17,1729.17,199.17,2179.17,
     52431.17, 2791.17,3295.17,3889.17,4357.17,4663.17,4861.17,
     6-3.566203E-3,0.,5.486467E-4,1.536211E-3,0.,1.097293F-3,-2.194587F-
     73,0.,0.,1.6459E-3,2.743233E-3,5.486467E-3,10.97293E-3,8.2297E-3,
     85.486467E-3,3.840527E-3,2.743233E-3,2.194587F-3,1.810534E-3,
     91.42648E-3,.9326993E-3,.365149E-3,0.,2116.2,471.5867,114.4748,
     A18.14354,2.302997,1.232752,.3814048,.02161063,.00436286,.0034331,
     B6.28109E-4,1.53598E-4,5.26658E-5,1.0571E-5,7.7157E-6,5.8324E-6,
     C3,5197E-6,1.4537E-6,3.9344E-7,8.4176E-8,2.2884E-8,7.2059E-9,0./
      REAL K,L,N,LHOE,MUO,LD,MUIN,MIN,LHXN,LEO
      COMMON FN,RHOEXN,RHOE
    1 READ (5,2) ALPHO
      READ(5,2) ALPHE(3), ALPHE(8), UOE(1), HOE,     TOE(1), PHI, ALT
      READ(5,2) ZOE(1),   HNNO,      PRO,     LEO, CD, LD,     UIN, STARTX
      READ(5,2) STOPX,    EXOUT
C     EXOUT=0.0 FOR DX=1.0 OUTPUT-------NOT=0.0 FOR DX=0.1 EXTRA OUTPUT
    2 FORMAT (8E10.8)
      FN=0.
      FN(2)=.33333333
      FN(3)=.66666666
      FN(4)=1.
      ALPHE=0.
      ALPHE(2)=0.
      ALPHE(4)=0.
      ALPHE(5)=0.
      ALPHE(6)=0.
```

```
      ALPHE(7)=0.
      DX=.1
      FNN=1.
      X=STARTX
      MM=1
      NN2=0
      NN3=0
      HH(1)=0.
      DO 1002 KK=2,23
 1002 HH(KK)=-.5*(AA(KK,2)-AA(KK-1,2))/(AA(KK,1)-AA(KK-1,1))          EQUATION 77
      BB=-HH(2)-HH(23)                                                EQUATION 80
      CC=(AA(1,2)+AA(23,2))/2.                                        EQUATION 79
      DD=CC+HH(2)*AA+HH(23)*AA(23,1)                                  EQUATION 78
      IF(ALT-295272.)1003,1003,1004
 1003 ALTF=(20855490.*ALT)/(20855490.+ALT)
      GO TO 1005
 1004 ALTF=ALT
 1005 FF=0.
      DO 1006 KK=2,22
      EE=HH(KK)-HH(KK+1)                                              EQUATION 81
 1006 FF=EE*((ALTF-AA(KK,1))*(ALTF-AA(KK,1)))**.5+FF
      TEMPM=DD+BB*ALTF+FF
      WO=28.9644
      IF(ALT-305114.4)1007,1007,1008
 1007 W=28.9644
      GO TO 1009
 1008 AKM=ALT/3280.8333
      W=32.154+AKM*(-.034513+AKM*(2.0326E-6+2.1032E-8*AKM))          EQUATION 74
 1009 TEMP=TEMPM*W/WO                                                 EQUATION 75
      IF(ALT-295272.)1010,1010,1016
 1010 DO 1011 KK=2,23
      IF(ALTF-AA(KK,1))1012,1011,1011
 1011 CONTINUE
 1012 KK=KK-1
      DALT=ALTF-AA(KK,1)
      IF(AA(KK,3))1013,1014,1015
 1013 PRESS=AA(KK,4)/(AA(KK,2)/TEMP)**(.0187434/(-AA(KK,3)))
      GO TO 1019
 1014 V1=(.0187434/AA(KK,2))*DALT
      PRESS=AA(KK,4)*(1./EXP(V1))
      GO TO 1019
 1015 PRESS=AA(KK,4)*(AA(KK,2)/TEMP)**(.0187434/AA(KK,3))
      GO TO 1019
 1016 DO 1017 KK=10,23
```

```
      IF(ALT-AA(KK,1))1018,1017,1017
 1017 CONTINUE
 1018 KK=KK-1
      DALT=ALT-AA(KK,1)
      XXX=S/(S-AA(KK,2)/AA(KK,3)+AA(KK,1))
      Y1=XXX*(ALOG((AA(KK,2)/AA(KK,3)+DALT)/(S+ALT))-ALOG((AA(KK,2)/
     1AA(KK,3))/(S+AA(KK,1))))
      Y2=-S*DALT/((S+AA(KK,1))*(S+ALT))
      Y3=XXX*(Y1+Y2)
      PRESS=AA(KK,4)/EXP(WO/AA(KK,3)*Y3/R)
 1019 DENS=PRESS/(53.352*TEMPM)
      G=GA*(S/(S+ALT))**2
      DENSG=DENS/32.174
      IF(ALT-295272.)1021,1021,1022
 1021 CS=49.1*TEMPM**.5
      SS=198.72
      BBB=7.3025E-7
      VIS=(BBB*TEMP**1.5)/(TEMP+SS)
      VISG=VIS/G
      MIN=UIN/CS
      RE=DENS*CS/VIS
      TEMPK=TEMP/1.8
      GO TO 1023
 1022 VISG=0.
      MIN=0.
      RE=0.
      TEMPK=0.
      CS=0.
 1023 PIN=PRESS
      TIN=TEMPK
      RHIN=DENSG
      MUIN=VISG
      UOE(2)=UIN
      TOE(2)=TIN
      ZOE(2)=1.
      P=(.0665*MIN**2*LD/X+1.)*PIN                                 EQUATION 68
      DO 10231 I=1,2
C     --EQUATION OF STATE--
10231 RHOE(I)=3.23E-4/ZOE(I)*P/TOE(I)
      WRITE(6,101)
  101 FORMAT(1H1)
      WRITE(6,102)
  102 FORMAT(19H INITIAL CONDITIONS)
      WRITE(6,106)
```

```
106 FORMAT (3H0 X15X2HUO14X2HUE14X2HTO14X2HTE14X2HHO14X2HHE)
    WRITE (6,104)X,UOE,TOE,HOE
    WRITE(6,103)
103 FORMAT(11H0 ALPHAO(O)7X9HALPHAO(N)7X10HALPHAO(O2)6X10HALPHAO(NO)6X
   111HALPHAO(NO+)5X11HALPHAO(O2-)5X10HALPHAO(E-)6X10HALPHAO(N2))
    WRITE(6,104)ALPHO
104 FORMAT(8E16.8)
    WRITE(6,105)
105 FORMAT(11H0 ALPHAE(O)7X9HALPHAE(N)7X10HALPHAE(O2)6X10HALPHAE(NO)6X
   111HALPHAE(NO+)5X11HALPHAE(O2-)5X10HALPHAE(E-)6X10HALPHAE(N2))
    WRITE(6,104)ALPHE
    WRITE(6,107)
107 FORMAT(6H0 M(O)12X4HM(N)12X5HM(O2)11X5HM(NO)11X6HM(NO+)10X6HM(O2-)
   110X5HM(E-)11X5HM(N2))
    WRITE(6,104)FM
    WRITE(6,108)
108 FORMAT(6H0 EP1912X4HEP2012X4HEP2112X4HEP2212X2HCD14X2HLD14X4HUINF1
   12X4HTINF)
    WRITE(6,104)EP19,EP20,EP21,EP22,CD,LD,UIN,TIN
    WRITE(6,109)
109 FORMAT(6H0 PINF12X6HRHOINF10X5HMUINF11X4HMINF12X2HN114X2HN214X2HN3
   114X2HN4)
    WRITE(6,104)PIN,RHIN,MUIN,MIN,FN
    WRITE(6,110)
110 FORMAT(5H0 PHI13X3HALT13X2HZO14X2HZE14X4HRHOO12X4HRHOE12X4HHNNO12X
   13HPRO)
    WRITE (6,104) PHI,ALT,ZOE,RHOE,HNNO,PRO
    WRITE (6,1112)
1112 FORMAT (3H0 W15X2HRE14X2HCS)
    WRITE(6,104) W,RE,CS
    WRITE(6,111)
111 FORMAT(5H0 LEO13X5HSTOPX11X2HDX)
    WRITE(6,104)LEO,STOPX,DX
    WRITE(6,101)
    F90=90./57.295779+.0001
    PHI1=PHI/57.295779
    IF(PHI.NE.90.)GO TO 121
    S2B=1.
    GO TO 122
121 COE(4)=-COS(PHI1)**2/MIN**4
    COE(3)=(2.*MIN**2+1.)/MIN**4+(1.44+.4/MIN**2)*SIN(PHI1)**2
    COE(2)=-((MIN**2+2.)/MIN**2+1.4*SIN(PHI1)**2)
    COE(1)=1.
    CALL RTSOLV(COE,3,ROOTR,ROOTI)
    DO 112 I=1,3
    IF(ABS(ROOTI(I)).GT..0000001)GO TO 120
```

SOLUTION OF EQUATION 67

```
112 CONTINUE
    GO TO 115
120 S2B=1.
    GO TO 122
115 DO 114 I=1,2
    DO 114 J=2,3
    IF(ROOTR(I).LE.ROOTR(J))GO TO 114
    TMP=ROOTR(I)
    ROOTR(I)=ROOTR(J)
    ROOTR(J)=TMP
114 CONTINUE
    S2B=ROOTR(2)
122 IF(MIN.LE.8.)GO TO 116
    Y=EXP(2.30258509*(5.48+.11*MIN))                                  EQUATION 66
    GO TO 123
116 Y=EXP(2.30258509*(4.744+MIN*(.124+.00976*MIN)))                   EQUATION 65
123 XT=MUIN/RHIN*Y/UIN*(-5./MIN**2+6.*(1.+5./MIN**2)*MIN**2*S2B/(MIN**  EQUATION 64
   12*S2B+5.)*(6./(7.*MIN**2*S2B-1.))**(5./7.))
100 IF(X.GE.XT)GO TO 117
    MUO=3.05E-8*(TOE/(TOE+110.))                                      EQUATION 62
    GO TO 118
117 MUO=.02*SQRT(DELM2)*RHOE(2)*(UOE(2)-UOE)                          EQUATION 63
118 DO 9 I=1,2
    GO TO (3,5),I
  3 DO 4 J=1,8
  4 ALPH(J)=ALPHO(J)
    GO TO 7
  5 DO 6 J=1,8
  6 ALPH(J)=ALPHF(J)
  7 TMP1= 5.+3.*EXP(-228./TOE(I))+EXP(-326./TOE(I))
    TMP2=1.-EXP(-2274./TOE(I))
    TMP3=3.+2.*EXP(-11390./TOE(I))
    TMP4=TOE(I)**(-1.5)
    TMP5=SQRT(TOE(I))
    TMP6=1.-EXP(-3395./TOE(I))
    TMP7=1.-EXP(-2740./TOE(I))
    TMP8=1.+EXP(-178./TOE(I))
    TMP9=1.-EXP(-1728./TOE(I))
    XP=-59365./TOE(I)
    IF(XP.GE.-88..AND.XP.LE.88.)GO TO 8
    IF(XP)10,10,11
 10 XP=-88.
    GO TO 8
 11 XP=88.
```

```
 8 A(I)=1.003E18*RHOE(I)*TMP4*(5.*ALPH(1)/8.+ALPH(3)/16.+ALPH(2)/21.+        A OF PARAGRAPH 4
  1ALPH(8)/42.+ALPH(4)/45.)*(.2345*ALPH(3)*TMP5*EXP(XP)*TMP2/TMP3*
  2TMP1-RHOE(I)*ALPH(1)**2)
   XP=-113200./TOE(I)
   IF(XP.GE.-88..AND.XP.LF.88.)GO TO 12
   IF(XP)13,13,14
13 XP=-88.
   GO TO 12
14 XP=88.
12 B(I)=1.147E19*RHOE(I)*TMP4*(5.*ALPH(2)/28.+ALPH(8)/56.+ALPH(1)/96.        B OF PARAGRAPH 4
  1+ALPH(3)/192.+ALPH(4)/180.)*(3.747*ALPH(8)*TMP5*EXP(XP)*TMP6-RHOE(
  2I)*ALPH(2)**2)
   XP=-75310./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 15
   IF(XP)16,16,17
16 XP=-88.
   GO TO 15
17 XP=88.
15 C(I)=2.15E19*RHOE(I)*TMP4*(ALPH(4)/30.+ALPH(1)/16.+ALPH(2)/14.+           C OF PARAGRAPH 4
  1ALPH(3)/32.+ALPH(8)/28.)*(.234*ALPH(4)*TMP5*EXP(XP)*TMP7/TMP8*
  2TMP1-RHOE(I)*ALPH(2)*ALPH(1))
   D(I)=3.45E10*RHOE(I)*TMP5*EXP(-3120./TOE(I))*(.994*ALPH(4)*ALPH(1)        D OF PARAGRAPH 4
  1*EXP(-15945./TOE(I))/TMP2*TMP7/TMP8*TMP3/TMP1-ALPH(2)*ALPH(3))
   XP=-37890./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 18
   IF(XP)19,19,20
19 XP=-88.
   GO TO 18
20 XP=88.
18 E(I)=4.78E11*RHOE(I)*(16.04*ALPH(1)*ALPH(8)*EXP(XP)/TMP7*TMP6/TMP1         E OF PARAGRAPH 4
  1*TMP8-ALPH(2)*ALPH(4))
   XP=-32130./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 21
   IF(XP)22,22,23
22 XP=-88.
   GO TO 21
23 XP=88.
21 F(I)=1.696E24*RHOE(I)*TMP4*(8.04E-12*ALPH(1)*ALPH(2)*TOE(I)/TMP6*         F OF PARAGRAPH 4
  1EXP(XP)/TMP1-ALPH(5)*ALPH(7))
   XP=-107440./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 24
   IF(XP)25,25,26
25 XP=-88.
   GO TO 24
```

```
26 XP=88.
24 K(I)=1.765E30*RHOE(I)*TMP4*ALPH(7)*(1.88E-12*ALPH(4)*TOE(I)**1.5      K OF PARAGRAPH 4
  1/TMP6*EXP(XP)/TMP8*TMP7-RHOE(I)*ALPH(5)*ALPH(7))
   XP=-43000./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 27
   IF(XP)28,28,29
28 XP=-88.
   GO TO 27
29 XP=88.
27 L(I)=1.65E22*RHOE(I)*TOE(I)**(-2.5)*EXP(XP)*(16.15*ALPH(8)*ALPH(3)    L OF PARAGRAPH 4
  1*EXP(-21945./TOE(I))*TMP6/TMP3*TMP2/TMP7*TMP8-ALPH(4)**2)
   N(I)=4.83E9*RHOE(I)*ALPH(3)*TOE(I)*(2.805E11*RHOE(I)*ALPH(3)*           N OF PARAGRAPH 4
  1ALPH(7)*TMP4/TMP9*TMP2/TMP3-ALPH(6)*EXP(-5110./TOE(I)))
   XP=-102000./TOE(I)
   IF(XP.GE.-88..AND.XP.LE.88.)GO TO 30
   IF(XP)31,31,32
31 XP=-88.
   GO TO 30
32 XP=88.
30 PP(I)=3.44E13*RHOE(I)*(.98*ALPH(5)*ALPH(6)*EXP(340./TOE(I))*TMP6*     P' OF PARAGRAPH 4
  1TMP9/TMP7*TMP8/TMP2*TMP3-ALPH(4)*ALPH(3)*EXP(XP))
   PPTO(I)=A(I)+FM(1)/FM(4)*(C(I)-D(I)-E(I))-FM(1)/FM(5)*F(I)              EQUATION 45
   PPTN(I)=B(I)+FM(2)/FM(4)*(C(I)+D(I)+E(I))-FM(2)/FM(5)*F(I)              EQUATION 46
   PPTO2(I)=-A(I)+FM(3)/FM(4)*(D(I)-EP20*L(I)/2.)-FM(3)/FM(6)*(EP21*       EQUATION 47
  1N(I)-EP22*PP(I))
   PPTN2(I)=-B(I)-FM(8)/FM(4)*(E(I)+EP20*L(I)/2.)                          EQUATION 48
   PPTNO(I)=-(C(I)+D(I)+EP19*K(I))+E(I)+EP20*L(I)+FM(4)/FM(6)*EP22*        EQUATION 49
  1PP(I)
   PPTEM(I)=FM(7)/FM(5)*F(I)+FM(7)/FM(4)*EP19*K(I)-FM(7)/FM(6)*EP21*       EQUATION 51
  1N(I)
   PPTO2M(I)=EP21*N(I)-EP22*PP(I)                                           EQUATION 52
 9 PPTNOP(I)=F(I)+FM(5)/FM(4)*EP19*K(I)-FM(5)/FM(6)*EP22*PP(I)             EQUATION 50
   DO 321 I=1,4
   DO 311 J=1,8
311 ALPHXN(J,I)=ALPHO(J)+(ALPHE(J)-ALPHO(J))*(6.*FN(I)**2-8.*FN(I)**3+     EQUATION 32
  13.*FN(I)**4)
   UXN(I)=UOE+(UOE(2)-UOE)*(6.*FN(I)**2-8.*FN(I)**3+3.*FN(I)**4)           EQUATION 32
   HXN(I)=HOE+(HOE(2)-HOE)*(10.*FN(I)**3-15.*FN(I)**4+6.*FN(I)**5)+        EQUATION 33
  1HNNO*.5*(FN(I)**2-3.*FN(I)**3+3.*FN(I)**4-FN(I)**5)
321 LHXN(I)=HXN(I)-.5*UXN(I)**2                                             EQUATION 53
   DO 33 I=1,4
   TBN=TOE
   IM=1
35 IF(ABS(TBN).LE.39.) TBN=39.
   TMP1=EXP(2274./TBN)-1.
```

```
      TMP2=EXP(3395./TBN)-1.
      TMP3=EXP(2740./TBN)-1.
      TMP4=EXP(1728./TBN)-1.
      T2=(LHXN(I)-2800./TMP1*2274.*ALPHXN(3,I)-3200./TMP2*3395.*ALPHXN(8
     1,I)-2985./TMP3*2740.*ALPHXN(4,I)-2985./TMP2*3395.*ALPHXN(5,I)-
     22800./TMP4*1728.*ALPHXN(6,I)-1.663E8*ALPHXN(1,I)-3.62E8*ALPHXN(2,I
     3)-3.28E7*ALPHXN(4,I)-3.536E8*ALPHXN(5,I)+1.429E7*ALPHXN(6,I))/
     4(3.5*2800.*ALPHXN(3,I)+3.5*3200.*ALPHXN(8,I)+1.4E4*ALPHXN(1,I)+
     51.6E4*ALPHXN(2,I)+3.5*2985.*ALPHXN(4,I)+3.5*2985.*ALPHXN(5,I)+
     64.074E8*ALPHXN(7,I)+3.5*2800.*ALPHXN(6,I))
      TEST=ABS(T2-TBN)/ABS(T2)
      IF(TEST-.00001)33,33,36
   36 GO TO(341,342),IM
  341 TBNM1=TOE
      TBN=T2
      T1=T2
      IM=2
      GO TO 35
  342 SA=(T2-T1)/(TBN-TBNM1)
      SQ=SA/(SA-1.)
      TBNP1=SQ*TBN+(1.-SQ)*T2
      T1=T2
      TBNM1=TBN
      TBN=TBNP1
      GO TO 35
   33 TXN(I)=T2
      P=(.0665*MIN**2*LD/X+1.)*PIN                                     EQUATION 68
      DO 37 I=1,4
   37 RHOEXN(I)=P/(8.9517E4*TXN(I)*(ALPHXN(3,I)/32.+ALPHXN(8,I)/28.+  EQUATION 55
     1ALPHXN(1,I)/16.+ALPHXN(2,I)/14.+ALPHXN(4,I)/30.+ALPHXN(5,I)/30.+
     21820.*ALPHXN(7,I)+ALPHXN(6,I)/32.))
      GO TO (38,39),MM
   38 CALL IGPAT(0.,1.,.01,ANS,1)
      THETA=(RHIN/RHOE(2)*CD/16.*LD**2)/(1.-210.*(P-PIN)/(RHOE(2)*UOE(2)   EQUATION 73
     1**2*(1.-UOE/UOE(2))*(10.+11.*UOE/UOE(2)))*ANS)
   39 DELM2=210.*THETA/((1.-UOE/UOE(2))*(10.+11.*UOE/UOE(2)))         EQUATION 40
      GO TO(391,392),MM
  391 DELM21=DELM2
  392 PPTSBO(1)=PPTO(1)
      PPTSBE(1)=PPTO(2)
      PPTSBO(2)=PPTN(1)
      PPTSBE(2)=PPTN(2)
      PPTSBO(3)=PPTO2(1)
      PPTSBE(3)=PPTO2(2)
```

```
      PPTSBO(4)=PPTNO(1)
      PPTSBF(4)=PPTNO(2)
      PPTSBO(5)=PPTNOP(1)
      PPTSBF(5)=PPTNOP(2)
      PPTSBO(5)=PPTO2M(1)
      PPTSBE(6)=PPTO2M(2)
      PPTSBO(7)=PPTEM(1)
      PPTSBE(7)=PPTEM(2)
      PPTSBO(8)=PPTN2(1)
      PPTSBF(8)=PPTN2(2)
      DO 40 I=1,8
      DALPHO(I)=24./RHOE(2)*MUO/UOE*LEO/(DELM2*PRO)*(ALPHE(I)-ALPHO(I))        EQUATION 36
     1+PPTSBO(I)/UOE
   40 DALPHF(I)=PPTSBE(I)/UOE(2)                                                EQUATION 38
      DO 41 I=1,6
      YY=ALPHO(I)
      DO 42 II=1,4
      FK(II)=DX*(24./RHOE(2)*MUO/UOE*LEO/DELM2*(ALPHE(I)-YY)/PRO+PPTSBO(
     1I)/UOE)
      GO TO (43,43,44,42),II
   43 YY=ALPHO(I)+FK(II)/2.
      GO TO 42
   44 YY=ALPHO(I)+FK(II)
   42 CONTINUE
   41 ALPHO(I)=ALPHO(I)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
      ALPHO(7)=FM(7)*ALPHO(5)/FM(5)-FM(7)*ALPHO(6)/FM(6)
      SUM=0.
      DO 45 I=1,7
   45 SUM=SUM+ALPHO(I)
      ALPHO(8)=1.-SUM
      DO 46 I=1,8
      FK(1)=DX*PPTSBE(I)/UOE(2)
   46 ALPHE(I)=ALPHE(I)+FK(1)
      ALPHE(7)=FM(7)*ALPHE(5)/FM(5)-FM(7)*ALPHE(6)/FM(6)
      SUM=0.
      DO 47 I=1,7
   47 SUM=SUM+ALPHE(I)
      ALPHE(8)=1.-SUM
      DPDX=-.0665* PIN*MIN**2*LD/X**2                                           dp/dx
      YY=UOF
      XX=X
      DO 51 I=1,4
      FK(I)=DX*(24./RHOE(2)*MUO/YY*(UOE(2)-YY)/DELM2+.0665/XX**2* PIN/
     1RHOE*MIN**2/YY*LD)
```

```
      GO TO (52,52,53,51),I
   52 XX=X+DX/2.
      YY=UOF+FK(I)/2.
      GO TO 51
   53 XX=X+DX
      YY=UOF+FK(I)
   51 CONTINUE
      DUODX=FK(1)/DX                                              du0 / dx
      GO TO(511,512),MM
  511 UOE1=UOE
  512 UOE=UOE+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
      XX=X
      YY=UOE(2)
      DO 54 I=1,4
      FK(I)=.0665/RHOE(2)*PIN/YY*MIN**2/XX**2*LD*DX
      GO TO (55,55,56,54),I
   55 XX=X+DX/2.
      YY=UOF(2)+FK(I)/2.
      GO TO 54
   56 XX=X+DX
      YY=UOE(2)+FK(I)
   54 CONTINUE                                                    due / dx
      DUEDX=FK(1)/DX
      GO TO(541,542),MM
  541 UOE1(2)=UOE(2)
      HNNO1=HNNO
      HOE1(2)=HOE(2)
  542 UOE(2)=UOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
      YY=TOE(2)
      DO 57 I=1,4
      TMP1=2274./YY
      TMP2=3395./YY
      TMP3=2740./YY
      TMP4=1728./YY
      TMP5=EXP(TMP1)-1.
      TMP6=EXP(TMP2)-1.
      TMP7=EXP(TMP3)-1.
      TMP8=EXP(TMP4)-1.
      FK(I)=-DX*YY*(1.118E-5*UOF(2)/YY*DUEDX+(7./2.+TMP1/TMP5)/32.*
     1DALPHE(3)+(7./2.+TMP2/TMP6)/28.*DALPHE(8)+(7./60.+TMP3/(TMP7*30.)+
     2366./YY)*DALPHE(4)+(7./60.+TMP2/(30.*TMP6)+3950./YY)*DALPHE(5)+
     3(5./32.+1853./YY)*DALPHE(1)+(7./64.+TMP4/(32.*TMP8)-159.3/YY)*
     4DALPHE(6)+(5./28.+4040./YY)*DALPHE(2)+4550.*DALPHE(7))/(((TMP1/
     5TMP5)**2+TMP1**2/TMP5+7./2.)*ALPHE(3)/32.+((TMP3/TMP7)**2+TMP3**2
     6/TMP7+7./2.)*ALPHE(4)/30.+((TMP2/TMP6)**2+TMP2**2/TMP6+7./2.)*
     7(ALPHE(8)/28.+ALPHE(5)/30.)+((TMP4/TMP8)**2+TMP4**2/TMP8+7./2.)*
```

```
   8ALPHE(6)/32.+5./32.*ALPHE(1)+5./28.*ALPHE(2)+4550.*ALPHE(7))
    GO TO (58,58,59,57),I
 58 YY=TOF(2)+FK(I)/2.
    GO TO 57
 59 YY=TOF(2)+FK(I)
 57 CONTINUE
    DLTEDX=FK(1)/(DX*TOE(2))                                          dℓn Te/dx, EQUATION 58
    TOE(2)=TOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
    SUM=0.
    SUM1=0.
    DO 60 I=1,8
    SUM=SUM+DALPHE(I)/FM(I)
 60 SUM1=SUM1+ALPHE(I)/FM(I)
    YY=RHOE(2)
    DO 61 I=1,4
    FK(I)=DX*YY*(DPDX/P-DLTEDX-SUM/SUM1)
    GO TO (62,62,63,61),I
 62 YY=RHOE(2)+FK(I)/2.
    GO TO 61
 63 YY=RHOE(2)+FK(I)
 61 CONTINUE                                                          dℓn ρe/dx
    DLREDX=FK(1)/(DX*RHOE(2))
    GO TO(611,612),MM
611 RHOE1(2)=RHOE(2)
    HOE1=HOE
612 RHOE(2)=RHOE(2)+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
    CALL IGRAT(0.,1.,.01,ANS,1)
    DEL=SQRT(2.*DELM2*ANS)                                            EQUATION 72
    YY=THETA
    DO 64 I=1,4
    FK(I)=DX*(-YY*DLREDX+(2.*YY+DEL**2/2.-DELM2*(4.+UOE/UOE(2))/10.)/
   1RHOE(2)*DPDX/UOE(2)**2)
    GO TO (65,65,66,64),I
 65 YY=THETA+FK(I)/2.
    GO TO 64
 66 YY=THETA+FK(I)
 64 CONTINUE
    DTHDX=FK(1)/DX                                                    dθ/dx
    THETA=THETA+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
    GO TO (67,68),MM
 67 MM=2
    DELTAC=4.*(HOE1(2)-HOE1)*(349.*UOE1(2)+311.*UOE1)-HNNO1*(43.*UOE1(  EQUATION 44
   12)+23.*UOE1)
 68 HNNO=4.*(HOE(2)-HOE)*(349.*UOE(2)+311.*UOE)/(43.*UOE(2)+23.*UOE)-  EQUATION 43
```

```
     1RHOE1(2)/RHOE(2)*DELM21/DELM2*DELTAC/(43.*UOE(2)+23.*UOE)
      SUM=0.
      CALL HSUB(TOE,HSUBS)
      DO 69 I=1,8
   69 SUM=SUM+HSUBS(I)*ALPHE(I)
      YY=HOE
      DO 70 I=1,4
      FK(I)=2./RHOE(2)*MUO/(UOE*DELM2)*(HNNO/PRO+12.*(PRO-1.)/PRO*UOE*
     1(UOE(2)-UOE)+12.*(LEO-1.)/PRO*(SUM-YY+UOE**2/2.))*DX
      GO TO (71,71,72,70),I
   71 YY=HOE+FK(I)/2.
      GO TO 70
   72 YY=HOE+FK(I)
   70 CONTINUE
      HOE=HOE+(FK(1)+2.*FK(2)+2.*FK(3)+FK(4))/6.
      TBN=TOE
      IM=1
   75 TMP11=EXP(2274./TBN)-1.
      TMP22=EXP(3395./TBN)-1.
      TMP33=EXP(2740./TBN)-1.
      TMP44=EXP(1728./TBN)-1.
      T2=(HOE-.5*UOE**2-2800./TMP11*2274.*ALPHO(3)-3200./TMP22*3395.*
     1ALPHO(8)-2985./TMP33*2740.*ALPHO(4)-2985./TMP22*3395.*ALPHO(5)-
     22800./TMP44*1728.*ALPHO(6)-1.663E8*ALPHO -3.62E8*ALPHO(2)-3.28E7*
     3ALPHO(4)-3.536E8*ALPHO(5)+1.429E7*ALPHO(6))/(3.5*2800.*ALPHO(3)+
     43.5*3200.*ALPHO(8)+1.4E4*ALPHO+1.6E4*ALPHO(2)+3.5*2985.*ALPHO(4)+
     53.5*2985.*ALPHO(5)+4.074E8*ALPHO(7)+3.5*2800.*ALPHO(7))
      TEST=ABS(T2-TBN)/ABS(T2)
      IF(TEST-.00001)74,74,741
  741 GO TO(742,743),IM
  742 TBNM1=TOE
      TBN=T2
      T1=T2
      IM=2
      GO TO 75
  743 SA=(T2-T1)/(TBN-TBNM1)
      SQ=SA/(SA-1.)
      TBNP1=SQ*TBN+(1.-SQ)*T2
      T1=T2
      TBNM1=TBN
      TBN=TBNP1
      GO TO 75
   74 TOE=T2
```

```
      CALL HSUB(TOE(2),HSUBS)
      SUM=0.
      DO 76 I=1,8
   76 SUM=SUM+ALPHE(I)*HSUBS(I)
      HOE(2)=.5*UOE(2)**2+SUM
      RHOE=3.23E-4/ZOE*P/TOF
      X=STARTX+FNN*DX
      IF(EXOUT.EQ.0.0)GO TO 90
C     --- INTERMEDIATE OUTPUT IF EXOUT IS NOT EQUAL TO ZERO ---
      WRITE (6,119)
  119 FORMAT(21H0INTERMEDIATE RESULTS)
      WRITE(6,104) X, P, UOE, TOF, HOE
      WRITE(6,104) ALPHO
      WRITE(6,104) ALPHE
      WRITE(6,104) DALPHO
      WRITE(6,104) DALPHE
      WRITE(6,104) A,    B,    C,    D
      WRITE(6,104) E,    F,    K,    L
      WRITE(6,104) N,    MUO, PP,   PPTO
      WRITE(6,104) PPTN,  PPTO2,  PPTN2,  PPTNO
      WRITE(6,104) PPTEM, PPTO2M, PPTNOP
      WRITE(6,104) DPDX,    DUODX,  DUEDX,  DLTEDX, THETA, DELM2
      WRITE(6,104) DLREDX, DEL,     DTHDX,  DELTAC,  S2B,  XT,  RHOE
      WRITE(6,104) ROOTR(1),ROOTI(1),ROOTR(2),ROOTI(2),ROOTR(3),ROOTI(3)
      WRITE(6,104) COE(1),  COE(2),  COE(3),  COE(4),  UXN
      WRITE(6,104) HXN,  LHXN
      WRITE(6,104) ALPHXN
   90 CONTINUE
      FNN=FNN+1.
      NN2=NN2+1
      IF(NN2.NE.10) GO TO 77
      NN2=0
      WRITE (6,1060)
 1060 FORMAT(3H0 X15X1HP15X2HUO14X2HUE14X2HTO14X2HTE14X2HHO14X2HHE)
      WRITE(6,104)X,P,UOE,TOF,HOE
      WRITE(6,103)
      WRITE(6,104)ALPHO
      WRITE(6,105)
      WRITE(6,104)ALPHE
      NN3=NN3+1
      IF(NN3.NE.5)GO TO 77
      NN3=0
      WRITE(6,101)
   77 IF((X+.000001).GE.50.)GO TO 1
      GO TO 100
      END
```

END PROGRAM

```
$IBFTC A5      LIST
      SUBROUTINE HSUB(T,HSUBS)
C     --                                            SUBROUTINE HSUB
      DIMENSION HSUBS(8)
      XP=2274./T
      X=7./2.
      HSUBS(3)=2800.*T*(X+XP/(EXP(XP)-1.))
      XP=3395./T
      HSUBS(8)=3200.*T*(X+XP/(EXP(XP)-1.))
      HSUBS(1)=1.4E4*T+1.663E8
      HSUBS(2)=1.6E4*T+3.62E8
      XP=2740./T
      HSUBS(4)=2985.*T*(X+XP/(EXP(XP)-1.))+3.28E7
      XP=3395./T
      HSUBS(5)=2985.*T*(X+XP/(EXP(XP)-1.))+3.536E8
      HSUBS(7)=4.074E8*T
      XP=1728./T
      HSUBS(6)=2800.*T*(X+XP/(EXP(XP)-1.))-1.429E7
      RETURN
      END
$IBFTC A4      LIST
      SUBROUTINE INTEQS(XX,YY,NOEQ)
C     --*                                           SUBROUTINE INTEQS
      COMMON FN(4),RHOEXN(4),RHOE(2)
      NOEQ=NOEQ
      CALL INTERP(XX,FN,RHOEXN,4,4,ANS,NERR)
      YY=RHOE(2)*XX/ANS
      RETURN
      END
$IBFTC A1      LIST
      SUBROUTINE IGRAT(LLIM,ULIM,DELTA,ANS,NOEQ)
C     --*                                           SUBROUTINE IGRAT
C                          1 LLIM  LOWER LIMIT OF INTEGRATION(FLOATING PT.)
C                          2 ULIM  UPPER LIMIT OF INTEGRATION
C                          3 DELTA STEP SIZE FOR SOLUTION
C                          4 ANS   VALUE OF INTEGRAL
C                          5 NOEQ  ALL EQUATIONS TO BE INTEGRATED MUST BE
C                                  COMPILED IN A SUBROUTINE CALLED INTEQS.
C                                  NOEQ IS TO BE USED BY THIS SUBROUTINE
C                                  TO SELECT THE PROPER EQUATION FOR
C                                  INTEGRATION IE. NOEQ=1 WILL BE TO INTEGRAT
C                                  EQUATION 1, NOEQ=2 TO INTEGRAT EQUATION
C                                  2, ETC. THE FORM OF INTEQS MUST BE...
C                                      INTEQS(X,Y,NOEQ)  WHERE
C                                        1 X     INDEPENDENT VARIABLE
C                                        2 Y     DEPENDENT VAR. IE. Y=F(X)
```

```
C                                  3 NOEQ SAME AS ABOVE. PROBABLY
C                                      USED IN A COMPUTED GO TO.
C                                              10 SEPT 64
C                                              RALPH SELLERS
      DIMENSION F(6),U(6),R(3)
      DATA (U(I), I=1,6) / .11930959,-.11930959,.33060469,-.33060469,
     A .46623476,-.46623476 /
      DATA (R(I), I=1,3) / .23395697,.18038079,.85662246E-1
      REAL LLIM
    5 A = LLIM
      DEL = DELTA
      LAST = 1
      ANS = 0.0
      IF(ABS (ULIM-LLIM) - .00001)80,80,10
   10 B = A + 4.0*DEL
      IF(B-ULIM)40,30,20
   20 B = ULIM
   30 LAST = 2
   40 DO 50   I=1,6
      X = (B-A)*U(I) + .5*(A+B)
      CALL INTEQS(X,F(I),NOEQ)
   50 CONTINUE
   60 ANS = ANS + (B-A)*(R(1)*(F(1)+F(2)) + R(2)*(F(3)+F(4)) + R(3)*
     A (F(5)+F(6)))
      GO TO (70,80),LAST
   70 A = B
      GO TO 10
   80 RETURN
      END
$IBFTC A2       LIST
      SUBROUTINE INTERP(ARG,XTAB,YTAB,NX,NP,ANS,NERR)
C     --                                       SUBROUTINE INTERP
C                    1. ARG  THE INDEPENDENT VARIABLE FOR THE DESIRED
C                            UNKNOWN.
C                    2. XTAB TABLE OF INDEPENDENT VALUES. MUST BE IN
C                            INCREASING ORDER.
C                    3. YTAB TABLE OF DEPENDENT VALUES.
C                    4. NX   NUMBER OF POINTS IN XTAB.
C                    5. NP   NUMBER OF POINTS FOR INTERPOLATION.
C                    6. ANS  THE DEPENDENT VALUE CORRESPONDING TO THE
C                            VALUE OF ARG.
C                    7. NERR WILL BE SET UNEQUAL ZERO IF ARG IS NOT ON
C                            XTAB. ROUTINE WILL EXTRAPOLATE.
C                                              10 SEPT 64
C                                              RALPH SELLERS
```

```
      DIMENSION XTAB(NX),YTAB(NX)
      NERR = 0
      IH = NP/2
      I = 1
      IF (XTAB(I) - ARG)30,20,10
   10 IH = 0
   12 NERR = 01
      GO TO 70
   20 ANS = YTAB(I)
      GO TO 999
   30 I = NX
      IF (XTAB( ) - ARG)12,20,50
   50 L = IH + 1
      DO 60   I=L,NX
      IF (XTAB(I) - ARG)60,20,70
   60 CONTINUE
   70 K = I - IH
      N = K + NP - 1
      ANS = 0.0
      IF (N - NX)90,90,80
   80 N = NX
      K = NX-NP+1
   90 DO 120  J=K,N
      P = 1.0
      DO 110  I=K,N
      IF (I-J)100,110,100
  100 P = P * (ARG - XTAB(I)) / (XTAB(J) - XTAB(I))
  110 CONTINUE
  120 ANS = ANS + YTAB(J) * P
  999 RETURN
      END
$IBFTC A3       LIST
      SUBROUTINE RTSOLV (COE,N1,ROOTR,ROOTI)
C     --                                                SUBROUTINE RTSOLV
      DIMENSION COE(51),ROOTR(50),ROOTI(50)
      N2=N1 + 1
      CTEBB=COE(1)
      DO 1 I=1,N2
    1 COE(I)=COE(I)/CTEBB
      AN3OD=ABS(COE(1))
      DO 2 J=2,N2
      IF(ABS(COE(J)).GT.AN3OD) AN3OD=ABS(COE(J))
      FJD=FLOAT(J)
      AKD=1.0/(FJD-1.0)
```

```
      TMD=AN3OD**AKD
    2 CONTINUE
      DO 3 K=1,N2
      J=K-1
    3 COE(K)=COE(K)/(TMD**J)
      N4=0
      I=N1+1
   19 IF(COE(I))9,7,9
    7 N4=N4+1
      ROOTR(N4)=0.
      ROOTI(N4)=0.
      I=I-1
      IF(N4-N1)19,37,19
    9 CONTINUE
   10 AXR=0.8
      AXI=0.
      L=1
      N3=1
      ALP1R=AXR
      ALP1I=AXI
      M=1
      GO TO 99
   11 BET1R=TEMR
      BET1I=TEMI
      AXR=0.85
      ALP2R=AXR
      ALP2I=AXI
      M=2
      GO TO 99
   12 BET2R=TEMR
      BET2I=TEMI
      AXR=0.9
      ALP3R=AXR
      ALP3I=AXI
      M=3
      GO TO 99
   13 BET3R=TEMR
      BET3I=TEMI
   14 TE1=ALP1R-ALP3R
      TE2=ALP1I-ALP3I
      TE5=ALP3R-ALP2R
      TE6=ALP3I-ALP2I
      TEM=TE5*TE5+TE6*TE6
      TE3=(TE1*TE5+TE2*TE6)/TEM
```

```
      TE4=(TE2*TE5-TE1*TE6)/TEM
      TE7=TE3+1.
      TE9=TE3*TE3-TE4*TE4
      TE10=2.*TE3*TE4
      DE15=TE7*BET3R-TE4*BET3I
      DE16=TE7*BET3I+TE4*BET3R
      TE11=TE3*BET2R-TE4*BET2I+BET1R-DE15
      TE12=TE3*BET2I+TE4*BET2R+BET1I-DE16
      TE7=TE9-1.
      TE1=TE9*BET2R-TE10*BET2I
      TE2=TE9*BET2I+TE10*BET2R
      TE13=TE1-BET1R-TE7*BET3R+TE10*BET3I
      TE14=TE2-BET1I-TE7*BET3I-TE10*BET3R
      TE15=DE15*TE3-DE16*TE4
      TE16=DE15*TE4+DE16*TE3
      TE1=TE13*TE13-TE14*TE14-4.*(TE11*TE15-TE12*TE16)
      TE2=2.*TE13*TE14-4.*(TE12*TE15+TE11*TE16)
      TEM=SQRT(TE1*TE1+TE2*TE2)
      IF(TE1)113,113,112
  113 TE4=SQRT(.5*(TEM-TE1))
      TE3=.5*TE2/TE4
      GO TO 111
  112 TE3=SQRT(.5*(TEM+TE1))
      IF(TE2)110,200,200
  110 TE3=-TE3
  200 TE4=.5*TE2/TE3
  111 TE7=TE13+TE3
      TE8=TE14+TE4
      TE9=TE13-TE3
      TE10=TE14-TE4
      TE1=2.*TE15
      TE2=2.*TE16
      IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
  204 TE7=TE9
      TE8=TE10
  205 TEM=TE7*TE7+TE8*TE8
      TE3=(TE1*TE7+TE2*TE8)/TEM
      TE4=(TE2*TE7-TE1*TE8)/TEM
      AXR=ALP3R+TE3*TE5-TE4*TE6
      AXI=ALP3I+TE3*TE6+TE4*TE5
      ALP4R=AXR
      ALP4I=AXI
```

```
      M=4
      GO TO 99
   15 N6=1
   38 IF(ABS(HELL)+ABS(BELL)-1.E-20)18,18,16
   16 TE7=ABS(ALP3R-AXR)+ABS(ALP3I-AXI)
      IF(TE7/(ABS(AXR)+ABS(AXI))-1.E-7)18,18,17
   17 N3=N3+1
      ALP1R=ALP2R
      ALP1I=ALP2I
      ALP2R=ALP3R
      ALP2I=ALP3I
      ALP3R=ALP4R
      ALP3I=ALP4I
      BET1R=BET2R
      BET1I=BET2I
      BET2R=BET3R
      BET2I=BET3I
      BET3R=TEMR
      BET3I=TEMI
      IF(N3-100)14,18,18
   18 N4=N4+1
      ROOTR(N4)=ALP4R
      ROOTI(N4)=ALP4I
      N3=0
   41 IF(N4-N1)30,37,37
   37 DO 5 L=1,N2
      ROOTR(L)=ROOTR(L)*TMD
    5 ROOTI(L)=ROOTI(L)*TMD
      DO 6 M=1,N2
      J=M - 1
    6 COE(M)=(COE(M)*(TMD**J))*CTEBB
      RETURN
   30 IF(ABS(ROOTI(N4))-1.E-5)10,10,31
   31 GO TO(32,10),L
   32 AXR=ALP1R
      AXI=-ALP1I
      ALP1I=-ALP1I
      M=5
      GO TO 99
   33 BET1R=TEMR
      BET1I=TEMI
      AXR=ALP2R
      AXI=-ALP2I
      ALP2I=-ALP2I
      M=6
```

```
      GO TO 99
   34 BET2R=TEMR
      BET2I=TEMI
      AXR=ALP3R
      AXI=-ALP3I
      ALP3I=-ALP3I
      L=2
      M=3
   99 TEMR=COE(1)
      TEMI=0.0
      DO 100 I=1,N1
      TE1=TEMR*AXR-TEMI*AXI
      TEMI=TEMI*AXR+TEMR*AXI
  100 TEMR=      TE1+COE(I+1)
      HELL=TEMR
      RELL=TEMI
   42 IF(N4)102,103,102
  102 DO 101 I=1,N4
      TEM1=AXR-ROOTR(I)
      TEM2=AXI-ROOTI(I)
      TE1=TEM1*TEM1+TEM2*TEM2
      TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
      TEMI=(TEMI*TEM1-TEMR*TEM2)/TE1
  101 TEMR=TE2
  103 GO TO(11,12,13,15,33,34),M
      END
```

Appendix B

SAMPLE CALCULATIONS

Sample cases involving a blunt body (ϕ_N = 90 degrees) and a cone (ϕ_N = 20 degrees) were run at 150 kilofeet and M_∞ = 17.2. The calculations began at x = 3 and carried through x = 50. As an example, the figure compares the axial temperature as a function of x for the blunt body and the cone. The transition from laminar to turbulent flow occurs sooner for the blunt body thereby initiating a rapid decay in axial temperature.

Input data for the cone are listed below. Velocities are in feet per second. Temperature is in degrees Kelvin. Enthalpies are in feet^2 per second^2. Altitude, diameter, and x are in feet. EXOUT determines the amount of output data. For minimum output, EXOUT = 0.0 or is omitted. If EXOUT $\neq$ 0.0, intermediate results at each Δx = 0.1 are printed as listed preceding statement 90 in Appendix A.

	Column							
Card	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
1	$a_0^{(O)}$	$a_0^{(N)}$	$a_0^{(O_2)}$	$a_0^{(NO)}$	$a_0^{(NO^+)}$	$a_0^{(O_2^-)}$	$a_0^{(e^-)}$	$a_0^{(N_2)}$
	0.232	0.0132	6.96 E-4	6.08 E-3	1.86 E-5	0.0	3.41 E-10	0.748
2	$a_e^{(O_2)}$	$a_e^{(N_2)}$	$(u_0)_c$	H_0	H_e	T_0	ϕ_N	ALT
	0.232	0.768	13,079.0	1.81646 E8	1.81646 E8	4030.0	20.0	150,000.0
3	Z_0	Hnn_0	Pr_0	Le_0	C_D	d	u_∞	X_c
	1.217	0.0	1.0	1.0	0.104	1.0	18,900.0	3.0
4	Stop X	EXOUT						
	50.0	0 0						

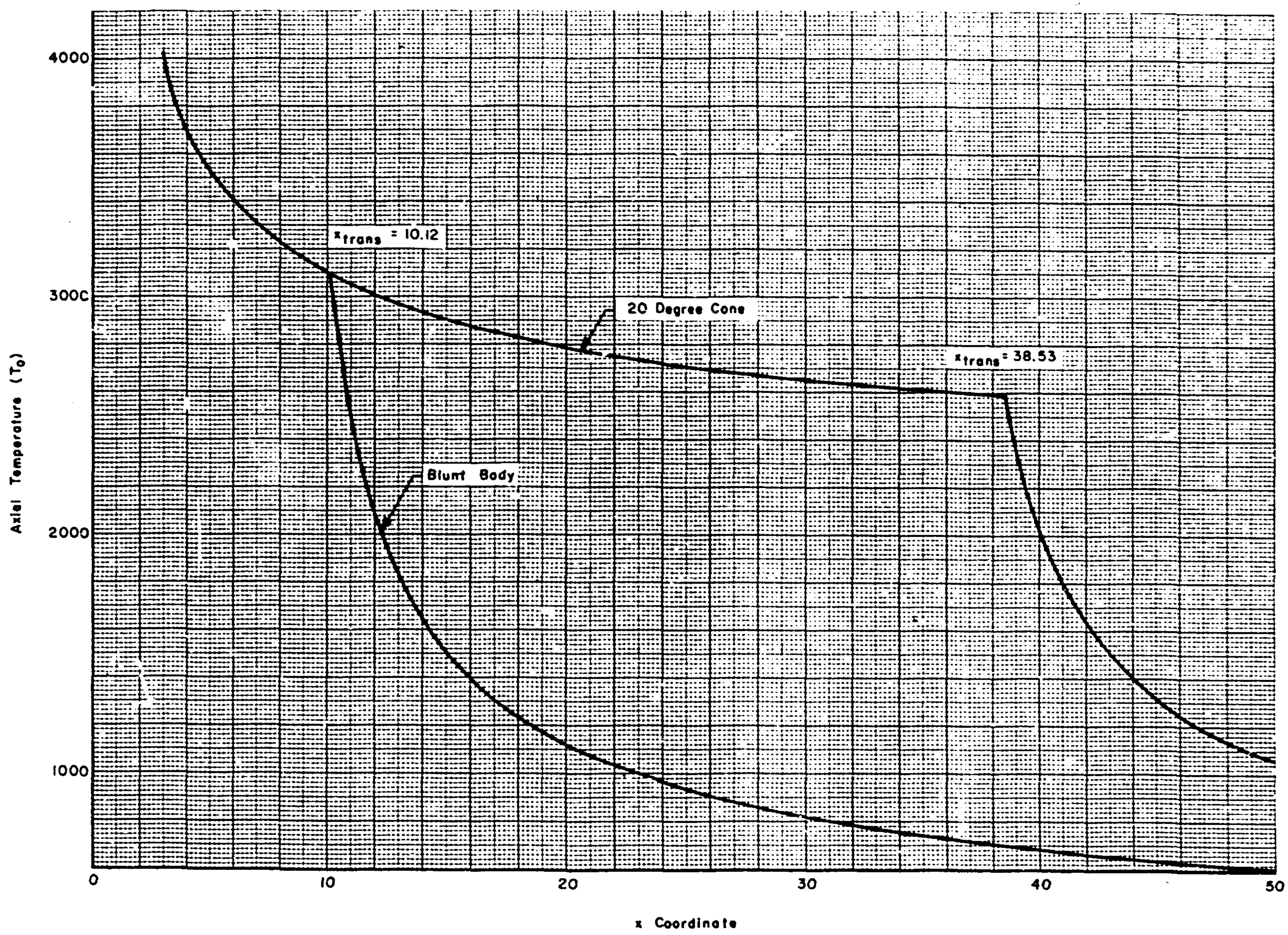

Figure. Axial Temperature Profile (°K)

OUTPUT DATA FOR A 20-DEGREE CONE -- PARTIAL LISTING

```
INITIAL CONDITIONS

X                UO               UE               TC               TE               HC               HE
0.30000000E 01   0.13079000E 05   0.18900000E 05   0.40300000E 04   0.26615319E 03   0.18164600E 09   0.18164600E 09

ALPHAO(O)        ALPHAO(N)        ALPHAO(O2)       ALPHAO(NO)       ALPHAO(NO+)      ALPHAO(O2-)      ALPHAO(E-)       ALPHAO(N2)
0.23200000E 00   0.13200000E-01   0.69600000E-03   0.60800000E-02   0.18600000E-04   0.               0.34100000E-09   0.74810000E 00

ALPHAE(O)        ALPHAE(N)        ALPHAE(O2)       ALPHAE(NO)       ALPHAE(NO+)      ALPHAE(O2-)      ALPHAE(E-)       ALPHAE(N2)
0.               0.               0.23200000E 00   0.               0.               0.               0.               0.76800000E 00

M(O)             M(N)             M(O2)            M(NO)            M(NO+)           M(O2-)           M(E-)            M(N2)
0.16000000E 02   0.14000000E 02   0.32000000E 02   0.30000000E 02   0.30000000E 02   0.32000000E 02   0.54171181E-04   0.28000000E 02

EP19             EP20             EP21             EP22             CD               LD               UINF             TINF
0.10000000E 01   0.10000000E 01   0.10000000E 01   0.10000000E 01   0.10399999E 00   0.10000000E 01   0.18900000E 05   0.26615319E 03

PINF             RHOINF           MUINF            MINF             N1               N2               N3               N4
0.28440321E 01   0.34583946E-05   0.35620289E-06   0.17586452E 02   0.               0.33333333E 00   0.66666666E 00   0.10000000E 01

PHI              ALT              ZO               ZE               RHOO             RHOE             HNNC             PRC
0.20000000E 02   0.15000000E 06   0.12170000E 01   0.10000000E 01   0.14713997E-05   0.27114065E-04   0.               0.10000000E 01

W                RE               CS
0.28964400E 02   0.10584838E 05   0.10746909E 04

LEO              STCPX            CX
0.10000000E 01   0.50000000E 02   0.09999999E 00

0.39999999E 01   0.17842537E 02   0.13338638E 05   0.18910281E 05   0.36873570E 04   0.24773294E 03   0.18164596E 09   0.18149366E 09

ALPHAO(O)        ALPHAO(N)        ALPHAO(O2)       ALPHAO(NO)       ALPHAO(NO+)      ALPHAO(O2-)      ALPHAO(E-)       ALPHAO(N2)
0.23306227E 00   0.12251321E-01   0.40063645E-03   0.46395113E-02   0.20730687E-04   0.45378821E-13   0.37433526E-10   0.74962553E 00

ALPHAE(O)        ALPHAE(N)        ALPHAE(O2)       ALPHAE(NO)       ALPHAE(NO+)      ALPHAE(O2-)      ALPHAE(E-)       ALPHAE(N2)
0.28692018E-35   0.15273701E-31   0.23200000E 00   0.53797533E-35   0.               0.               0.               0.76800000E 00

X                P                UO               UE               TO               TE               HO               HE
0.49999999E 01   0.14781618E 02   0.13519584E 05   0.18917510E 05   0.35262995E 04   0.23485822E 03   0.18164593E 09   0.18149030E 09

ALPHAO(O)        ALPHAO(N)        ALPHAO(O2)       ALPHAO(NO)       ALPHAO(NO+)      ALPHAO(O2-)      ALPHAO(E-)       ALPHAO(N2)
0.23377801E 00   0.11608448E-01   0.27701030E-03   0.35280336E-02   0.21634841E-04   0.31814575E-13   0.39066163E-10   0.75078686E 00

ALPHAE(O)        ALPHAE(N)        ALPHAE(O2)       ALPHAE(NO)       ALPHAE(NO+)      ALPHAE(O2-)      ALPHAE(E-)       ALPHAE(N2)
0.67743083E-35   0.29176994E-31   0.23200000E 00   0.12701828E-34   0.               0.               0.               0.76800000E 00

X                P                UO               UE               TO               TE               HO               HE
0.59999999E 01   0.12758298E 02   0.13655374E 05   0.18922940E 05   0.34031580E 04   0.22521913E 03   0.18164588E 09   0.18148820E 09
```

X	P	UO	UE	TO	TE	HO	HE
ALPHAO(O)	ALPHAO(N)	ALPHAO(O2)	ALPHAO(NO)	ALPHAO(NO+)	ALPHAO(O2-)	ALPHAO(E-)	ALPHAO(N2)
0.23426126E 00	0.11170580E-01	0.20670090E-03	0.27530917E-02	0.21923756E-04	0.22341484E-13	0.39587858E-10	0.75158644E 00
ALPHAE(O)	ALPHAE(N)	ALPHAE(O2)	ALPHAE(NO)	ALPHAE(NO+)	ALPHAE(O2-)	ALPHAE(E-)	ALPHAE(N2)
0.97365092E-35	0.42125975E-31	0.23200000E 00	0.18255955E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.69999999E 01	0.11321448E 02	0.13761912E 05	0.18927205E 05	0.33050843E 04	0.21766693E 03	0.18164585E 09	0.18148677E 09
ALPHAO(O)	ALPHAO(N)	ALPHAO(O2)	ALPHAO(NO)	ALPHAO(NO+)	ALPHAO(O2-)	ALPHAO(E-)	ALPHAO(N2)
0.23460232E 00	0.10858068E-01	0.16190571E-03	0.21971828E-02	0.21931233E-04	0.16202855E-13	0.39601319E-10	0.75215858E 00
ALPHAE(O)	ALPHAE(N)	ALPHAE(O2)	ALPHAE(NO)	ALPHAE(NO+)	ALPHAE(O2-)	ALPHAE(E-)	ALPHAE(N2)
0.11865397E-34	0.54363545E-31	0.23200000E 00	0.22247619E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.79999999E 01	0.10248358E 02	0.13848211E 05	0.18930664E 05	0.32246682E 04	0.21155480E 03	0.18164581E 09	0.18148574E 09
ALPHAO(O)	ALPHAO(N)	ALPHAO(O2)	ALPHAO(NO)	ALPHAO(NO+)	ALPHAO(O2-)	ALPHAO(E-)	ALPHAO(N2)
0.23485154E 00	0.10626556E-01	0.13172096E-03	0.17862590E-02	0.21797786E-04	0.12105054E-13	0.39360393E-10	0.75258213E 00
ALPHAE(O)	ALPHAE(N)	ALPHAE(O2)	ALPHAE(NO)	ALPHAE(NO+)	ALPHAE(O2-)	ALPHAE(E-)	ALPHAE(N2)
0.13414066E-34	0.66046275E-31	0.23200000E 00	0.23151374E-34	0.	0.	0.	0.76800000E 00
⋮							
X	P	UO	UE	TO	TE	HO	HE
0.49000000E 02	0.40402319E 01	0.17724082E 05	0.18958918E 05	0.10952071E 04	0.16183112E 03	0.18148260E 09	0.18148023E 09
ALPHAO(O)	ALPHAO(N)	ALPHAO(O2)	ALPHAO(NO)	ALPHAO(NO+)	ALPHAO(O2-)	ALPHAO(E-)	ALPHAO(N2)
0.69008109E-01	0.31645631E-04	0.16160735E 00	0.46615661E-02	0.36367753E-05	0.19062316E-09	0.65666243E-11	0.76468769E 00
ALPHAE(O)	ALPHAE(N)	ALPHAE(O2)	ALPHAE(NO)	ALPHAE(NO+)	ALPHAE(O2-)	ALPHAE(E-)	ALPHAE(N2)
0.21115285E-34	0.42227179E-30	0.23200000E 00	0.39591153E-34	0.	0.	0.	0.76800000E 00
X	P	UO	UE	TO	TE	HO	HE
0.50000000E 02	0.40162599E 01	0.17785059E 05	0.18959076E 05	0.10549995E 04	0.16155300E 03	0.18148222E 09	0.18148019E 09
ALPHAO(O)	ALPHAO(N)	ALPHAO(O2)	ALPHAO(NO)	ALPHAO(NO+)	ALPHAO(O2-)	ALPHAO(E-)	ALPHAO(N2)
0.65683020E-01	0.21694389E-04	0.16499310E 00	0.44480612E-02	0.34021324E-05	0.22784827E-09	0.61428651E-11	0.76485072E 00
ALPHAE(O)	ALPHAE(N)	ALPHAE(O2)	ALPHAE(NO)	ALPHAE(NO+)	ALPHAE(O2-)	ALPHAE(E-)	ALPHAE(N2)
0.21155599E-34	0.42997908E-30	0.23200000E 00	0.39666741E-34	0.	0.	0.	0.76800000E 00